STUDY OF SEDIMENT TRANSPORT RATE OVER DUNE-COVERED BEDS

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Abstract

River bed configuration, as a roughness element, exerts flow resistance and influences sediment transport process. In this study, a movable bed experiment has been carried out to evaluate the influence of form drag, exerted by microscale bedforms, on sediment transport rate. The experimental results have been compared with the bedload transport rate calculated by using Ashida & Michiue's and Meyer-Peter & Muller's formulae. The bedload transport rate, measured directly in the present experiments, appears to be in good agreement with the bedload transport estimated by both relationships. Furthermore, experimental result has been compared with the result of bedload transport calculated by using a numerical model proposed by Giri & Shimizu. This numerical model incorporates a stochastic pick up-deposition model for non-equilibrium sediment transport with a distribution function of mean step-length proposed by Nakagawa & Tsujimoto. It is revealed that the mean step-length has an effect on sediment transport rate, thereby exerts influence on bedform geometry, particularly on wave-length. Consequently, a basic attempt has been made to calibrate parameter mean step-length based on experimental result and numerical analysis. Comparison between experimental and numerical simulation results shows reasonably good agreement for both the bedload transport rate and geometric characteristics of bedforms.

Key Words : *dune, bedload transport rate, sediment transport rate, bedform*

1. INTRODUCTION

The underlying mechanism of resistance to flow exerted by microscale bedforms, particularly dunes, has been explored since long. Such microscale sand waves have dominant effect in flow resistance and so this is the issue of a great practical significance in river engineering. Estimates of bedload transport rate considering such bedforms are important to predict channel evolution process. This information is necessary for river planning and restoration. On the other hand, flow resistance due to the form drag exerted by bedforms is important to quantify its influence on water surface fluctuation. The flow, the bed morphology, and the sediment-transport field are tightly coupled and none may be predicted without knowledge of the others.

A comprehensive review of past investigations on river dunes has been presented by Best (2005). A variety of approaches have been developed in order to analyze bedform-induced morphology in alluvial channels and the development of dunes (Engelund, 1970; Engelund & Fredsøe, 1982; Coleman & Melville, 1994; Fredsøe, 1996; Coleman & Melville, 1996; Lisle et al., 1997; Baas, 1999; Smith, 1970; Fredsøe, 1974; Kobayashi & Madsen, 1985; McLean & Smith, 1986; Nelson & Smith, 1989). Most of them are based on linear stability theory. Some works were conducted using weakly nonlinear stability analysis (Ji & Mendoza, 1997; Yamaguchi & Izumi, 2005). Some notable examples of field observations include those of Itakura et al. (1986), Kishi and Kuroki (1972), Dinehart (1989), and Carling et al. (2000a, 2000b). All these studies have revealed the influence of temporal variability contributing to bed form development.

Recently, a significant contribution has been made by Giri & Shimizu (2006), which demonstrates the capability of a numerical model to reproduce fluid and morphodynamic features of dunes. Proposed model is vertical two-dimensional with non-hydrostatic free surface flow, which is coupled with a stochastic pick up-deposition model (Nakagawa & Tsujimoto, 1980) for non-equilibrium sediment transport by imposing a distribution function of mean step-length. This numerical model can simulate formation and migration process of dunes and physically replicate geometric characteristics of dunes for arbitrary steady or unsteady flow condition. The model can predict form drag exerted by dunes hydrodynamically.

The microscale sand waves are categorized into three distinctive configurations, namely lower regime, upper regime and transition. In this study, we have focused on bedform evolution and sediment transport process under the lower regime. We have made a basic attempt to measure sediment transport rate as well as bedform geometry in laboratory flume under variety of flow conditions. We have conducted series of movable bed experiments to evaluate quantitatively sediment transport rate with bedforms and associated form drag. We compared the experimental results with the bedload transport rate calculated by using a couple of well-known bedload transport formulae, and also with the result of numerical calculation by using above-mentioned morphodynamic model.

2. MOVABLE BED EXPERIMENT

(1) Experimental procedure

There are few movable bed experiments with bedforms that pay attention to the sediment transport rate. Some significant set of experiments has been compiled by Guy et al. (1966). Some laboratory experiments were carried out to evaluate sediment transport rate under unsteady flow condition (De Sutter et al., 2001; Lee et al., 2004). The primary purpose of this study is to estimate the sediment transport rate simultaneously conducting observation on temporal bed deformation under steady flow condition with lower regime, and finally verify the results against numerical computation.

Experimental works reported in this paper have been carried out in the Hydraulic Research Laboratory of Hokkaido University. Total length of flume was 10m and height was 30cm. Experiments was performed for two different flume widths, namely 10cm and 15cm. The flume was made from Plexiglas. The middle part of the flume (7.5m in case of flume with 10cm width and 8m in case of flume with 15cm width) was covered with a 5 cm thick sediment layer, which was trapped using acrylic sediment stoppers at the upstream and downstream part of the flume in order to prevent the degradation of the channel bed. A tail gate was used in order to control flow-depth and keep the mean velocity under nearly uniform condition.

The experimental setup also included bed load trap to collect bedload sediment transport. The slope of the channel bed was set to be 0.002. Well-sorted uniform sediment with 0.335mm median diameter (D_{50}) was used for all experimental case. The grain size distribution curve is depicted in **Fig. 1**. The sand feeding was done manually by filling up the space between movable bed and sand stopper in the upstream boundary so as to maintain constant amount of the sediment feeding throughout the experiment.

Moreover, the bed elevation at the both upstream and downstream ends was carefully controlled throughout the experimental run. Initially, a preliminary development of bed was allowed, and a uniform flow was set up through the adjustment of tail gate to make the mean water surface slope nearly equal to the bed slope. After sufficiently long period of time, an equilibrium stage was assumed to have achieved. Bed configuration and water-depth were recorded after achieving the equilibrium condition, the sediment transport rate were measured three times with an interval of 30 minutes. The sediment transport rate was evaluated by taking an average of the three measured values. The migration process of sand dune was visualized using a digital camera. Geometric characteristics of dunes (wave-length and wave-height) and migration rate in each experiment were estimated by taking average value of all representative dunes after achieving the dynamic equilibrium conditions.

The hydraulic conditions for these experiments are presented in **Table 1**.

Fig. 1 Grain size distribution curve.

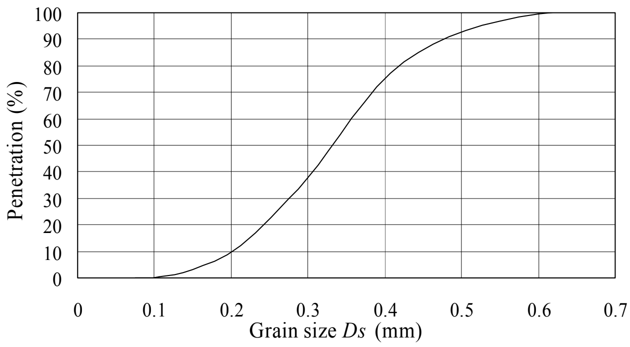

Table 1 Experimental condition.

Run No.	Channel Width(cm)	Discharge(cm^2/sec)	Slope	Grain Size(mm)
A-1		207.69		
A-2	10	221.05		
A-3		355.18		
A-4		439.70		
B-1		224.23	0.002	0.28
B-2		305.51		
B-3	15	335.48		
B-4		410.14		
B-5		414.09		

(2) Dune-induced bedload transport

The sediment transport rate measured in experiments was compared with the bedload transport rate calculated by using Ashida & Michiue's and Meyer-Peter & Muller's formulae. It is well-known that the average flow-depth increases when dunes appear on the flat-bed as the total flow resistance becomes larger due to the form drag exerted by the bed configuration. Following the principle of Einstein & Barbarossa (1952), Kishi & Kuroki (1972) suggested that the total flow resistance is associated with the skin friction of sediment particles and the form drag exerted by bed configuration. Consequently, the total flow resistance is expressed as follows:

$$\tau_* = \tau'_* + \tau''_* \tag{1}$$

where τ_* = dimensionless bed shear stress, τ'_* = dimensionless effective shear stress, τ''_* = dimensionless form drag.

The skin friction is generally assumed as the effective shear stress, which is usually used to calculate sediment transport rate. However, this may not be true in the presence of bedforms. The dimensionless total flow resistance and effective shear stress is expressed as follows (Kishi & Kuroki, 1972):

$$\tau_* = \frac{h I}{R_S \, d} \tag{2}$$

$$\tau'_* = \frac{h' I}{R_S \, d} \tag{3}$$

where I = bed slope, R_S = relative density of sediment in water, namely 1.65, d = grain size diameter, h = total flow-depth, h' = flow-depth, associated with effective shear stress.

The velocity, associated with effective shear stress can be calculated as (Kishi & Kuroki, 1972):

$$\overline{U} = \frac{q}{h} = \sqrt{gh'I}\left[6.0 + 5.75 \log_{10}\left(\frac{h'}{k_s}\right)\right] \tag{4}$$

where $\overline{U}$ = average flow-velocity, q = discharge per unit width, g = gravity acceleration, k_s = equivalent roughness height.

The equivalent roughness height can be calculated as $k_s = md$, in which m is generally proposed to be 2.5. Consequently, the sediment transport rate can be expressed as follows:

$$q_B = q_{B*}\sqrt{R_S g d}\, d \tag{5}$$

where q_{B*} = dimensionless sediment transport rate.

In this study, we applied the effective shear stress given by Equation (3) to bedload formulae (Equation 6 and 7 below) instead of the bed shear stress in order to estimate the sediment transport rate over the bed configuration observed in the present experiments.

The dimensionless sediment transport rate is calculated using Ashida & Michiue's (1972) and Meyer-Peter & Muller's formulae repectively as:

$$q_{B*} = 17(\tau_*)^{3/2}\left(1-\sqrt{\frac{\tau_{c*}}{\tau_*}}\right)\left(1-\frac{\tau_{c*}}{\tau_*}\right) \tag{6}$$

$$q_{B*} = 8(\tau_*)^{3/2}\left(1-\frac{\tau_{c*}}{\tau_*}\right)^{3/2} \tag{7}$$

where τ_{c*} = dimensionless critical tractive force.

The dimensionless critical tractive force is estimated using the Iwagaki's empirical equation:

$$\tau_{c*} = 0.0052 d^{-21/32} \tag{8}$$

For, $0.0065(cm) \le d \le 0.0565(cm)$

(3) Experimental results

The dimensionless sediment transport rate versus dimensionless effective shear stress is plotted in **Fig. 2**. In this figure, the symbols show the experimental value while each line represents the bedload transport rate estimated by using Ashida & Michiue's and Meyer-Peter & Muller's formulae. According to the **Fig. 2**, the bedload transport rate, which was measured directly in the present experiments, shows good agreement with the bedload rate estimated by both the bedload formulae. It is found that the bedload rate on dune-covered bed can be estimated by using these conventional bedload transport formulae using effective shear stress instead of grain shear stress. Furthermore, we evaluated geometric characteristics of dunes for different flow intensity. **Fig. 3** shows the relationship between wavelength and unit discharge. It is clear from **Fig. 3** that the wavelength of dune tends to be longer with increasing unit discharge. Likewise, **Fig. 4** shows the wave height versus unit discharge relation. It was found that wave height in the case of 10cm wide flume is higher than in the case of 15cm wide flume for similar unit discharge.

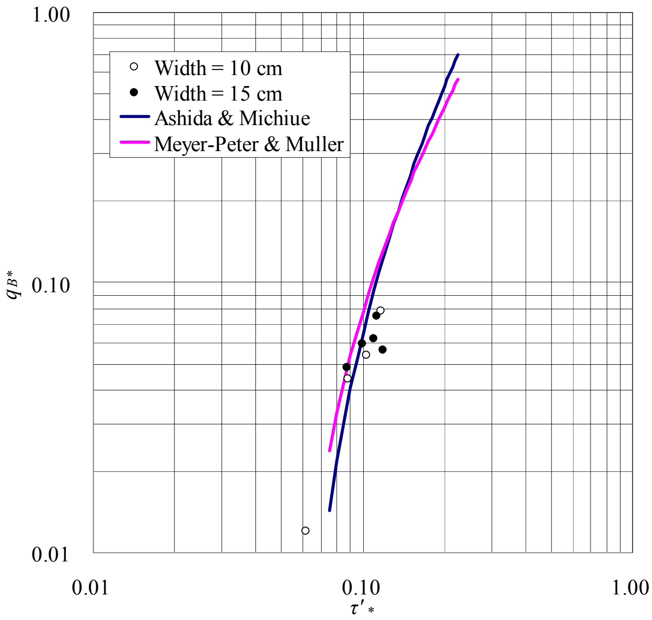

Fig. 2 Non-dimensional bedload transport versus bed shear stress: Comparison between bedload transport formulae and the measurement.

Fig. 5 depicts the relationship between dune migration rate and unit discharge. It can be seen that the dune migration velocity tends to become faster when the discharge increases. Also, we can see from the result that migration rate is lower in case of narrow flume for low flow intensity whereas higher for high flow intensity in comparison with the case of wider flume. The inconsistency in the results between flumes with two different width can be attributed to the wall effect.

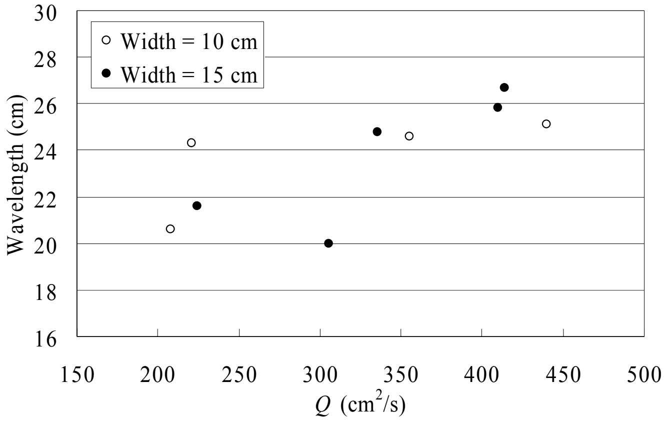

Fig. 3 Relationship between duen wave-length and unit discharge.

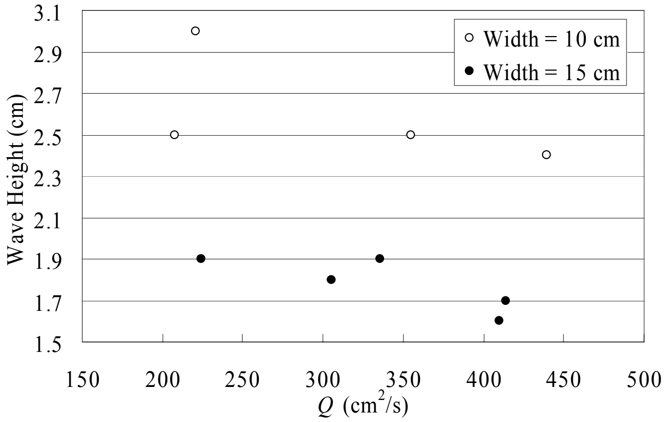

Fig. 4 Relationship between dune wave-height and unit discharge.

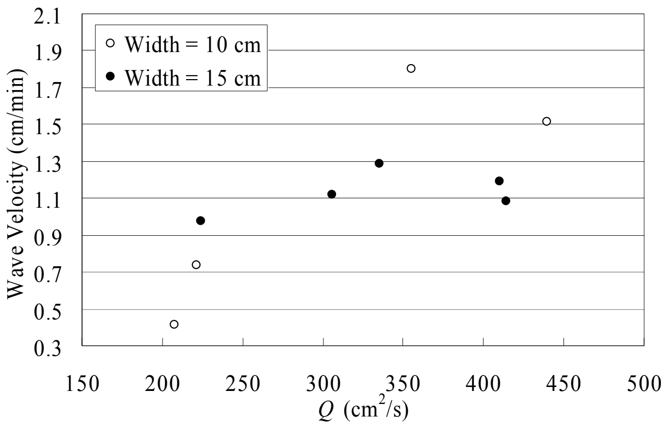

Fig. 5 Relationship between dune migration rate and unit discharge.

3. NUMERICAL CALCULATION

(1) Numerical calculation method

Giri & Shimizu (2006) performed the numerical calculation of bed deformation with a vertical two-dimensional model with non-hydrostatic free surface flow, coupled with an Eulerian stochastic formulation of sediment transport proposed by Nakagawa & Tsujimoto (1980). The proposed sediment transport computational approach explicitly considers the flow variability during morphodynamic computation. Consequently, the exchange of sediment particles, namely pickup and deposition rate at each time step, can be computed by coupling sediment transport with the hydrodynamic model. The phase lag between boundary shear stress and sediment transport is generated due to the imposition of mean step-length of sediment particle. The form drag exerted by dune is replicated hydrodynamically by the numerical model.

The rate of bed deformation can be computed using following sediment balance equation:

$$\frac{\partial y_b}{\partial t}+\frac{1}{1-\lambda}\left[\frac{A_3}{A_2}(p_d-p_s)d\right]=0 \tag{9}$$

where y_b = bed elevation, λ = porosity of sediment particle, A_2 and A_3 = shape coefficients of sand grain for two and three dimensional geometrical properties, respectively (= $\pi / 4$, $\pi / 6$).

The pick-up rate p_s is evaluated by using a relationship proposed by Nakagawa & Tsujimoto (1980). In the dimensionless form, pick up rate can be expressed as follows:

$$p_{s*}=p_s\sqrt{d/(\rho_s/\rho-1)g}=0.03\tau_*(1-0.035/\tau_*)^3 \tag{10}$$

where p_{s*} = dimensionless pick up rate, ρ_s and ρ = fluid and sediment density respectively.

The deposition rate p_d is calculated as a product of pick-up rate and the probability density distribution of mean step-length.

The sediment deposition rate reads as:

$$p_d=\int_0^{\infty}p_s(x-s)f_s(s)ds \tag{11}$$

where p_d = sediment deposition rate and f_s(s) = distribution function of mean step length.

Distribution function of mean step length is found to be exponential as follows (Nakagawa & Tsujimoto, 1980):

$$f_s(s)=\frac{1}{\Lambda}\exp\left(-\frac{s}{\Lambda}\right) \tag{12}$$

where Λ = the mean step length and s = the distance of sediment motion from pick up point.

The mean step-length can be calculated as $\Lambda = \alpha d$, in which α is an empirical constant and proposed to be 100. However, we will later show that the dune wavelength depends on this parameter.

The bedload transport rate by using an Eulerian stochastic formulation can be calculated as follows:

$$q_B=q_{B*}\sqrt{R_S gd}\,d=\frac{A_3}{A_2}d\int_{-\infty}^{x}p_s(x')\int_{x-x'}^{\infty}f_s(s)dsdx' \tag{13}$$

The numerical model incorporates both the bedload and suspended sediment transport, but in this study we considered bedload only. Also, we performed numerical calculation for one of the cases, namely run A-2, within the scope of this study.

4. COMPARISON BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

We performed numerical computation and evaluated the effect of empirical parameter α that is used to calculate mean step-length. We performed a couple of cases with α equal to 20 and 50, i.e. the step length was assumed to be 20 and 50 times of the particle diameter respectively, in order to evaluate the effect of mean step-length on bed configuration and sediment transport rate.

Fig. 6 shows the instantaneous bed configurations of experiment and numerical simulation in 7200 seconds. Result revealed that wave-length and wave-height of the dune reproduced by the numerical model in the case of means step-length equal to 20*d* were 18cm and 2.1cm respectively, while wave-length and wave-height of the dune in the case of 50*d* were 24cm and 2.5cm respectively, and wave-length and wave height of the dune observed in experiment were 24.3cm and 3cm respectively. It can be seen that the wave-length of the dune reproduced by the numerical model is longer in case of higher value of mean step-length (50*d*) than that of in case of lower value (20*d*). Consequently, for the calculation conditions presented herein, the dune formation estimated by numerical model appears to be depending on the mean step-length. Regarding the migration rate, the average value was found to be 2.020 cm/min in case of mean step-length equal to 50*d* which is nearer to observation. However, average migration rate of dunes seems to be overpredicted in comparison with experimental value. It is found that wave-length and wave height of dunes estimated by the numerical calculation are in good agreement with measured values in the present experiments, when the mean step-length is assumed to be 50*d*.

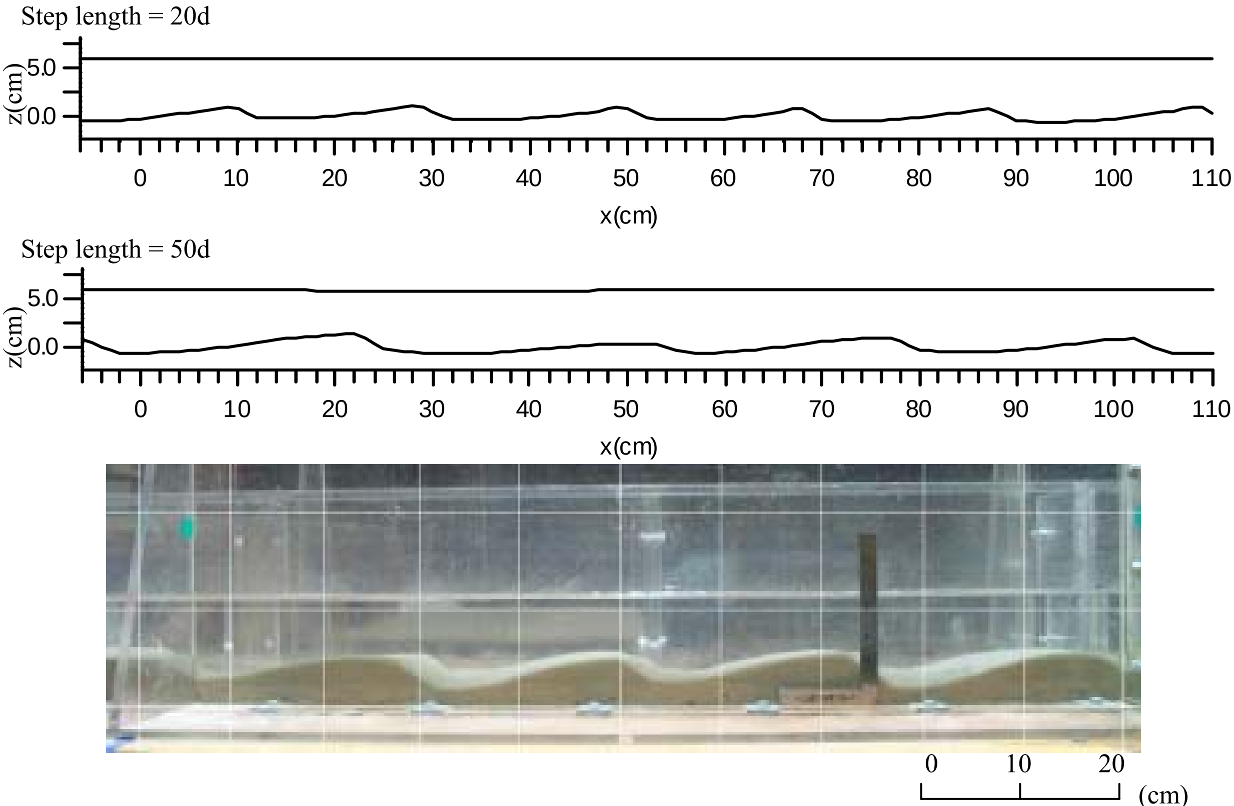

Fig. 6 The bed configurations of experiment and numerical simulation 7200 seconds from water run.

Fig. 7 shows the dimensionless sediment transport rate and also the effect of mean step-length depicted in two different plots. In the same figure, the upper line shows the water-depth, and the lower line shows the dimensionless sediment transport rate. Here also the dimensionless sediment transport rate in the case of 50*d* seems to be larger than in the case of 20*d*.

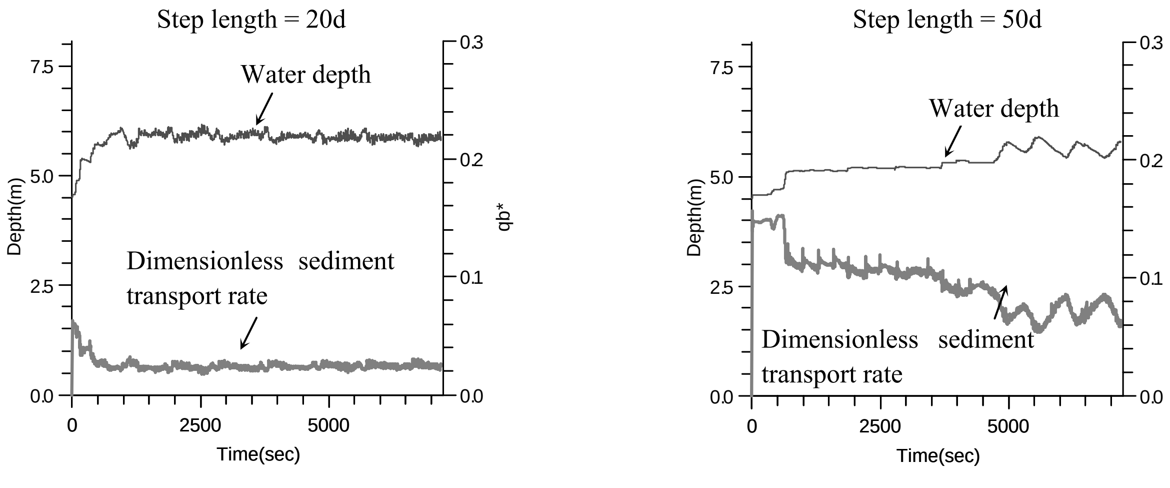

Fig. 7 The water depth and the dimensionless sediment transport rate in the case of each step-length.

Finally, **Fig. 8** shows the relationship between dimensionless effective shear stress and dimensionless bedload transport rate, and comparison between numerical simulation and the experimental measurement. In numerical calculation, the dimensionless effective shear stress and the dimensionless bedload transport were evaluated by taking an average value within the range of 5400 second to 7200 second. We compared calculated bedload transport rate with experimental measurement using both value of mean step-length (20*d* and 50*d*). From the result, it can be observed that the experimental value of effective shear stress is lower than both numerical calculation results. On the other hand, bedload transport rate seems to be higher in experiment in comparison with numerical result when used mean step-length equal to 20*d*, whereas it appears lower in case of step-length equal to 50*d*. The comparison shows that the numerical calculation of bedload transport with step-length equal to 50*d* is nearer to experimental value. Some inconsistency is noticed, which can be attributed to the side wall effect on sediment transport rate.

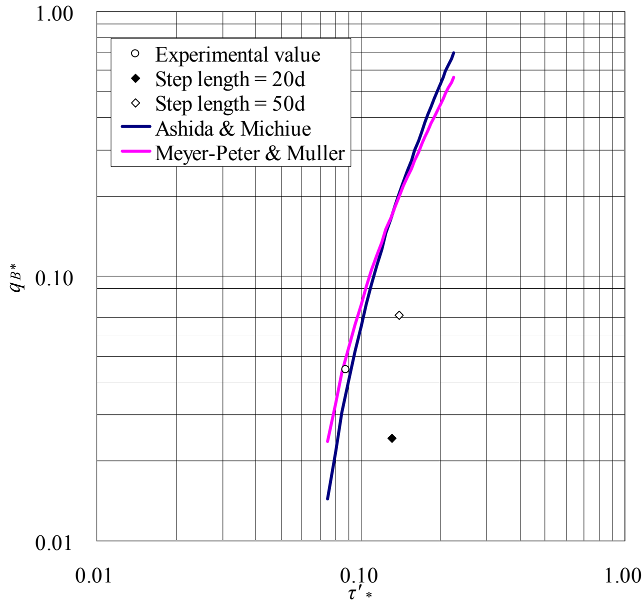

Fig. 8 Relationship between dimensionless effective shear stress and dimensionless bedload transport rate: Comparison between numerical calculation and the experiment.

5. CONCLUSIONS

In this study, we conducted experiments on bedload transport induced by dunes. We performed comparison of bedload transport rate between laboratory observation, calculated values using Ashida & Michiue's and Meyer-Peter & Muller's formulae, and the results of numerical calculation by using a two-dimensional morphodynamic model. The bedload transport rate measured directly in the present experiments shows good agreement with the bedload rate estimated by both the bedload formulae. However, it is necessary to simply consider the effective shear stress instead of grain shear stress in these conventional bedload transport formulae. The numerical simulation of dune evolution was found to be depending on the mean step-length, an empirical sediment parameter used in pick up-deposition model for non-equilibrium sediment transport. As a result of the comparison between observation and numerical calculation, it is found that the dune formation calculated by numerical model shows good agreement with the observed one in case when the mean step-length is employed as 50*d*. The wavelength of dune tends to decrease in the calculation when the mean step-length is assumed to be smaller than 50*d*. The bedload transport rate estimated by the numerical calculation also shows good agreement with the measured value in the present experiments when the step-length is assumed to be 50*d*. It implies that both the bed deformation and sediment transport can be calculated by the numerical simulation with the reasonable value of mean step-length. The influence of mean step-length and its physical determination is supposed to be explored more comprehensively.

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